SIMILARITY OF FLOWS OF SPONTANEOUSLY CONDENSING GAS IN SUPERSONIC NOZZLES

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UDC 533.6.671:533.6.011.3

Questions of the similarity of flows of relaxing media are of particular importance in view of the extreme complexity of such processes. The traditional method of obtaining the dimensionless combinations — the similarity parameters — can be rather ineffective in this case. Such a situation arises when a large number of criteria have to be satisfied simultaneously, which often makes accurate modeling of effects impossible. In view of this a search for approximate or partial modeling conditions, or for various correlations, may be more fruitful.

Examples of relaxing media are two-phase media, whose general flow similarity conditions were obtained in [1]. This system of parameters was used in conjunction with empirical data on some features of the effect in [2, 3] for the case of spontaneous condensation associated with steady and unsteady flows of supercooled vapor. The semiempirical similarity formulated in [2, 3] can be refined if the question of the boundary conditions and nozzle shapes that allow accurate modeling of flows is considered. On the other hand, the set of similar flows can be expanded by reconsidering the principle of entropy correlation of condensation shocks from the viewpoint of precise modeling [4].

1. Spontaneous condensation of a real gas flowing through nozzles is a complex phenomenon whose features have been inadequately investigated in a number of cases. Such cases include condensation in high-velocity, highly supercooled flows, condensation of a gas in which the internal degrees of freedom of the molecules are in a nonequilibrium state, condensation in a multicomponent medium, and others.

In view of this we confine ourselves to an analysis of the simplest and most widely used model, based on the Frenkel'—Zel'dovich theory.

We make the following assumptions:

1) The system is adiabatic;

2) the flow as a whole can be either steady or unsteady; there is no phase slip;

3) the condensing gas is thermally and calorically ideal;

4) nucleation is quasisteady;

5) the condensate is uniformly distributed in the gas phase;

6) the drops are spherical, their growth rate is independent of their size, their temperature is equal to the saturation temperature at the given pressure of the surrounding gas, and heat transfer between the phases can be neglected;

7) the temperature dependence of the surface tension coefficient is given by the Ramsay—Shields—Eötvös equation [1], and the density of the liquid phase and heat of vaporization are constant.

According to the above, for the description of a flow of two-phase medium in a channel of prescribed shape we can use the following system of dimensionless equations:

$$\mathrm{Sh}F\frac{\partial\rho}{\partial t}+\frac{\partial\rho uF}{\partial x}=0,\quad \mathrm{Sh}\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\mathrm{Eu}\frac{\partial p}{\partial x}=0, \tag{1.1}$$

$$\mathrm{Sh}\rho\frac{\partial}{\partial t}\left(e+\mathrm{Eu}\frac{p}{\rho}\right)+\rho u\frac{\partial}{\partial x}\left(e+\mathrm{Eu}\frac{p}{\rho}\right)=\mathrm{Sh}\,\mathrm{Eu}\frac{\partial p}{\partial t}+\mathrm{Eu}\,\rho u\frac{\partial p}{\partial x};$$

Zhukovskii. Translated from Zhurnal Prikladnoi Mekhaniki Tekhnicheskoi Fiziki, No. 1, pp. 75-83, January—February, 1982. Original article submitted December 16, 1980.

 0021-8944/82/2301-0068$07.50

$$\mathrm{Sh}\frac{\partial y}{\partial t}+u\frac{\partial y}{\partial x}=I_1\left(\frac{4}{3}\pi I_2^3\frac{J}{\rho}r_*^3+4\pi\dot{r}\Omega_2\right), \tag{1.2}$$

$$\mathrm{Sh}\frac{\partial\Omega_2}{\partial t}+u\frac{\partial\Omega_2}{\partial x}=I_3\left(I_2^2\frac{J}{\rho}r_*^2+2\dot{r}\Omega_1\right),$$

$$\mathrm{Sh}\frac{\partial\Omega_1}{\partial t}+u\frac{\partial\Omega_1}{\partial x}=I_3\left(I_2\frac{J}{\rho}r_*+\dot{r}\Omega_0\right),$$

$$\mathrm{Sh}\frac{\partial\Omega_0}{\partial t}+u\frac{\partial\Omega_0}{\partial x}=I_3\frac{J}{\rho};$$

$$F=\frac{F^0}{F_c}=f(x), \tag{1.3}$$

$$p=\rho_nT,\quad e=\frac{1}{\varkappa-1}T-yI_4I_5,\quad y=1-\frac{\rho_n}{\rho},$$

$$J=\rho_n^2(I_4-T)^{1/2}\exp\left[-\beta I_6r_*^2(I_4-T)\right],$$

$$r_*=\frac{I_4-T}{T\ln\frac{p}{p_s}},\quad \dot{r}=\alpha T^{1/2}\left[1-\left(\frac{T}{T_s}\right)^{1/2}\right],$$

$$\ln p_s(T)=I_4I_5(1-T^{-1}),$$

$$\mathrm{Sh}=\frac{l_c}{\tau u_{c0}},\quad \mathbf{Eu}=\frac{p_c}{\rho_cu_{c0}^2},\quad u_{c0}=(RT_c)^{1/2},$$

$$I_1=\frac{\rho_0J_cl_c^4}{\rho_cu_{c0}},\quad J_c=\frac{\rho_c^2}{\rho_0}\left(\frac{2\sigma_*N_A^3}{\pi\mu^3}\right)^{1/2},$$

$$I_2=\frac{r_c}{l_c}=\frac{2\sigma_*}{\rho_0Rl_c},\quad I_3=\frac{U}{u_{c0}},\quad U=\frac{p_c}{\rho_0(2\pi RT_c)^{1/2}},$$

$$I_4=\frac{T^*}{T_c},\quad I_5=\frac{L}{RT^*},\quad I_6=\frac{16\pi\sigma_*^3}{3\rho_0^2kR^2},$$

where ρ is the density of the two-phase medium; ρ_n is the density of the gas phase; p is the pressure; T is the temperature; u is the velocity; e is the internal energy; y is the degree of condensation; J is the nucleation rate; $\dot{r}$ is the drop growth rate; r_* is the radius of the condensation nucleus; $p_s(T)$ is the saturation pressure; F is the area of current jet; x is the coordinate; t is the time; τ is the characteristic time of the unsteady process; l_c is the characteristic length; ρ_0 is the density of the liquid phase; L is the heat of vaporization; R is the gas constant; k is the Boltzmann constant; $\varkappa$ is the adiabatic exponent; N^A is the Avogadro number; μ is the molecular weight; T^* is the critical temperature; σ_* is the coefficient in the Ramsay—Shields—Eötvös equation for the surface tension; α is the condensation coefficient; and β is a factor correcting the nucleation energy. The system of gasdynamic equations (1.1) contains similarity parameters: Sh is the Strouhal number and Eu is the Euler number. The system of equations for the phase-change kinetics (1.2) contains parameters I_{1-6}, which are associated with the spontaneous condensation process. The subscript c denotes the characteristic scales of the corresponding gasdynamic quantities. The parameter I_1 is the ratio of the characteristic condensation flow to the characteristic gasdynamic flow, I_2 is the ratio of the characteristic length of the condensation nucleus to the characteristic gasdynamic length l_c, I_3 is the ratio of the characteristic drop growth rate U to the characteristic gasdynamic velocity u_{c0}. The parameters $I_{5,6}$ depend on the thermophysical properties of the substance, and the role of parameter I_4 will become clear after the scales of the gasdynamic parameters have been defined.

The system of equations (1.1)-(1.3) becomes meaningful on attainment of the saturation state, in which the boundary and initial conditions for its solution should be prescribed. In the general case the solution of system (1.1)-(1.3) even with steady boundary conditions may be of a self-oscillating nature, due to the appearance of an unsteady shock wave [3]. When such a state is realized the characteristic scale t of the unsteady process will depend on the boundary conditions and it can be established only by obtaining the corresponding solution. In view of this we confine ourselves to an analysis of the steady-state approximation of system (1.1)-(1.3) at this stage.

We assume that at certain values p_c, T_c, ρ_c, and u_c°, prescribed in the saturation cross section F_c of a channel of variable cross section, the flow is adiabatic in the familiar gasdynamic sense. It is obviously convenient to choose the values p_c, T_c, and ρ_c

as scales for the gasdynamic parameters. The boundary conditions have the form $p = T = \rho = F = 1$, $y = \Omega_0 = \Omega_1 = \Omega_2 = 0$, $u_c = u_c^0/u_{c0}$, and $Eu = 1$.

For a specific substance the parameters $I_{5,6}$ are fixed, the parameter I_3 can be expressed in terms of I_4 and I_5, and from the conditions I_1 = idem, I_2 = idem we must have l_c = idem. As a result, the parameters l_c, I_4, u_c, α, and β will be independent. Within the framework of the considered spontaneous condensation theory the coefficients α and β are not defined. Available data indicate that they can depend both on the local flow parameters and on the entire prehistory of the flow of specific gas [2, 3, 5]. On the other hand, in certain ranges of the parameters the coefficients α and β vary slightly and, hence, we can assume that they are constant and exclude them from the characteristic parameters. As a result, the similarity conditions reduce to reproduction of the flow velocity at the saturation point u_c, the characteristic flow scale l_c, and the parameter I_4, and the variation of the dimensionless area of the channel (1.3) below the saturation cross section will be given by the same function of the dimensional coordinate x^0, i.e., the equalities

$$u_c = \text{idem},\ I_4 = \text{idem},\ l_c = \text{idem},\ f(x^0) = \text{idem}. \quad (1.4)$$

will be fulfilled. If the saturation state occurs downstream of the critical cross section of a Laval nozzle, then the entropy S_0 and the total enthalpy H_0 in the flow of superheated gas will be constant. The condition I_4 = idem is identical to the conditions

$$S_0 = \text{idem and } T_c = \text{idem}. \quad (1.5)$$

We take into account the relation $u_c = \left[2\frac{\varkappa}{\varkappa-1}\left(\frac{T_0}{T_c}-1\right)\right]^{1/2}$, where T_0 is the stagnation temperature. The condition u_c = idem is then identical to the condition T_0/T_c = idem, which in conjunction with (1.5) necessitates reproduction of the stagnation parameters T_0 and p_0. Thus, if l_c = idem and $f(x^0)$ = idem, only flows with fixed stagnation parameters can be similar.

It is obvious that the question of the choice of the characteristic length l_c when $f(x^0)$ = idem has to be solved in each specific case, proceeding from the flow characteristics. For instance, if flows in nozzles of limited extent are considered, the characteristic length selected should be the distance between the saturation cross section and exit cross section of the nozzle, with relation (1.3) valid in this length — in other words, nozzles in which similar flows can occur can differ only in transverse scale. Such rigid limitations rule out, in fact, the possibility of modeling flows of a spontaneously condensing gas. Nevertheless, we can assume that not all the parameters in (1.4) have an equal effect on the accuracy of flow reproduction [2]. In view of this we take into account the following fact.

We know that in the case of constant entropy (1.5) for nozzles with straight generatrices partial similarity exists in the hypersonic approximation within the framework of the entropy correlation principle [4]. Instead of reproducing the stagnation temperature T_0 with S_0 = idem we require the reproduction of the parameter

$$\varphi_1 = \frac{h_*}{\mathrm{tg}\gamma} T_0^{\frac{1}{i(\varkappa-1)} - \frac{1}{2}},$$

where h_* is the diameter of the critical cross section; 2γ is the nozzle angle; $i = 1$ and 2 for plane and axisymmetric flows, respectively.

In the given case the distributions of the gasdynamic functions on the downstream side of the saturation state will be similar if we use the following transformation of the coordinate x^0:

$$x = x^0 \left(\frac{h_*}{\tan\gamma} T_0^{\frac{1}{i(\varkappa-1)} - \frac{2}{3}}\right)^3.$$

Since nozzles with straight generatrices are widely used in practice it is of interest to assess the role of the condition T_0 = idem within the framework of exact similarity and to compare the latter with the entropy correlation. The characteristic gasdynamic scale l_c can conveniently be connected with the dimension of the saturation cross section F_c in the following way:

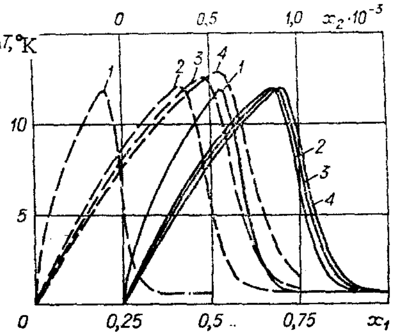

Fig. 1

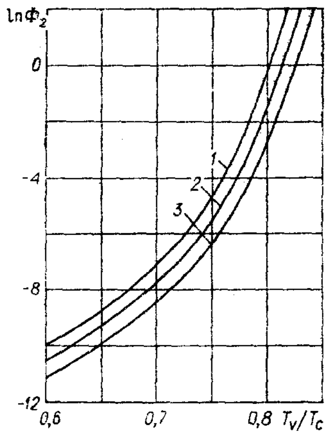

Fig. 2

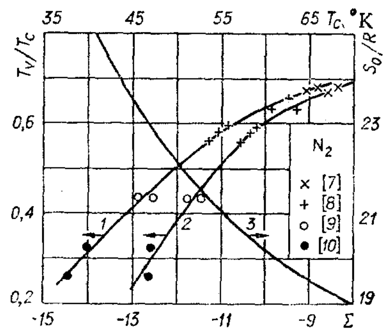

Fig. 3

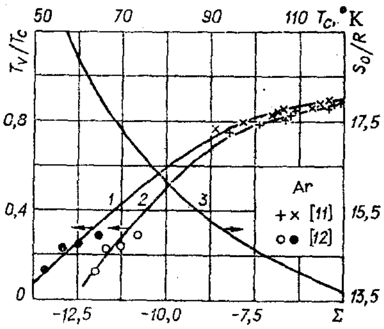

Fig. 4

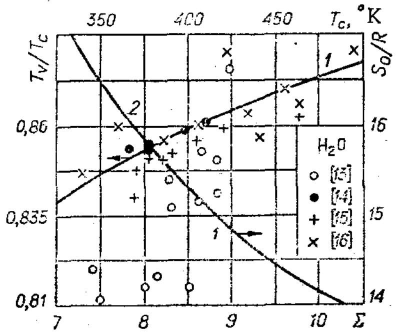

Fig. 5

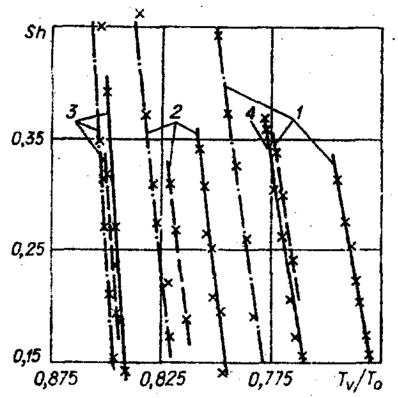

Fig. 6

$$l_c = \frac{F^{1/i}}{\pi^{1-\frac{1}{i}} b^{\frac{2}{i}-1} \tan\gamma},$$

where b is the width of the plane nozzle.

If the saturation state is attained downstream of the critical cross section, l_c will be uniquely connected with the diameter of the critical cross section h_* and the parameter $I_4 = T_0/T_c$

$$l_c = f(\varkappa, i)\frac{h_*}{\tan\gamma}\left(\frac{T_0}{T_c} - 1\right)^{-\frac{1}{2i}}\left(\frac{T_0}{T_c}\right)^{\frac{\varkappa+1}{2i(\varkappa-1)}}$$

or in the hypersonic approximation with $T_0/T_c \gg 1$

$$l_c \sim \varphi_2 = \frac{h_*}{\tan\gamma}\left(\frac{T_0}{T_c}\right)^{\frac{1}{i(\varkappa-1)}}.$$

Thus, for nozzles with straight generatrices at $M \gg 1$ the main similarity conditions have the form φ_2 = idem and S_0 = idem, and we will demonstrate the role of the condition T_0 = idem on the basis of parametric calculations.

The results of such calculations for nitrogen are shown in Fig. 1 in the form of distributions in x of the isobaric supercooling ΔT. As a reference flow we take a flow with $T_0 = 94$°K, $p_0 = 1.92 \cdot 10^5$ Pa ($I_4 = 1.22$) in a conical nozzle with geometric parameter $h_*/\tan\gamma = 1$ cm. The corresponding curves are marked with the number 1 in Fig. 1. Further, with increase in T_0 and S_0 = idem the geometric parameter of the nozzle was altered in accordance with the conditions φ_2 = idem and φ_1 = idem relative to the reference flow. The distribution of ΔT is shown in Fig. 1 for these cases by dashed and continuous lines, respectively. Variants 2-4 correspond to stagnation temperatures 200, 450, and 800°K and I_4 = 2.61, 5.88, and 10.42. For the dashed curves the x axis is directed downward, and $x_1 = x^0/\varphi_2$. For the continuous curves the x axis is directed upward, the origin is shifted to the right, and the coordinate X_2 is the product

$$x_2 = x^0\left[\frac{h_*}{\tan\gamma}\left(\frac{T_0}{T_c}\right)^{\frac{1}{i(\varkappa-1)}-\frac{2}{3}}\right]^3.$$

A comparison of the presented results shows the following. The entropy correlation leads to better reproduction of the maximum supercooling ΔT of the flow than in the first case, and stabilization of the distribution of the gasdynamic parameters occurs when $I_4 \geqslant 3$. In turn, within the framework of exact similarity the stagnation temperature has a significant effect on the distribution of the parameters, the position of the maximum supercooling point (Wilson point), and its value.

Thus, entropy correlation [4] is a more exact method of constructing engineering calculations than the known approximate similarity variant [2]. On the basis of [4] we can obtain a combined parameter for treatment of the experimental data for condensation shocks.

2. One of the components of the method of calculating the maximum supercooling of the flow within the framework of the entropy correlation [5, 6] is graphical solution of the equation for the Wilson point

$$\Phi(T_V, S_0) = \varphi_1\left(\frac{h_*}{\tan\gamma}, T_0\right) \tag{2.1}$$

(see relation (4) from [6]). For this the left-hand side of (2.1) is represented by a nomogram in the form of lines of equal values of entropy S_0 for different static temperatures at the Wilson point T_V. An example of such a nomogram is Fig. 1 in [6]. It is easy to see that the lines S_0 = const in this figure are similar in shape and are shifted through almost the same distance with the same change in entropy, and that their vertical asymptotes correspond to temperature $T_c(S_0)$.

In view of this the following transformation of (2.1) is of interest. We convert the thermodynamic parameters on the left-hand side of (2.1) to dimensionless form by analogy with Sec. 1, and we transfer the additional scale factors to the right-hand side. As a result we obtain

$$\Phi_1\left(\frac{T_V}{T_c}, S_0\right) = \varphi_3, \text{ where } \varphi_3 = \varphi_1 p_c T_c^{-\frac{1}{2}-\frac{1}{i(\varkappa-1)}}.$$

Taking into account the relation

$$\frac{S_0}{R} = \frac{\varkappa}{\varkappa-1}\ln T_c - \ln p_c + \frac{S'}{R}, \tag{2.2}$$

we can express the parameter φ_3 in the following way:

$$\ln\varphi_3 = \ln\varphi_1 + \left(\frac{\varkappa+1}{2(\varkappa-1)} - \frac{1}{i(\varkappa-1)}\right)\ln T_c - \frac{S_0 - S'}{R},$$

where S' is the entropy constant. We recall that the temperature T_c is determined by the entropy S_0. The degree of dependence of Φ_1 on S_0 can be established numerically.

For instance, Fig. 2 shows the results of calculation of Φ_1 in the form of lines of equal values of S_0 ($S_0 \cdot 10^{-3}$ J/(kg·°K) = 7, 6.1, 5.2 – curves 1-3, respectively) as functions of T_V/T_c; for the whole range of S_0 we used a single relation for the saturation line of the form

$$\log p_S = A - B/T. \tag{2.3}$$

The great congestion of the curves in the considered case in comparison with the nomograms from [6] indicates a weak dependence of Φ_1 on S_0. This fact can be used to represent the experimental results if the data on the condensation shocks in nozzles (or jets) are treated in the variables

$$\sum = \ln\varphi_1 + \left[\frac{\varkappa+1}{2(\varkappa-1)} - \frac{1}{i(\varkappa-1)}\right]\ln T_c - \frac{S_0}{R},\ \frac{T_V}{T_c}. \tag{2.4}$$

To determine the values of T_c it is desirable to use a relation of type (2.3).

The results of such treatment of condensation shocks in flows of nitrogen [7-10], argon [11, 12], and water vapor [13-16] are shown in Figs. 3-5, respectively. Curves 1 and 2, drawn through the experimental points in Figs. 3 and 4, correspond to plane and axisymmetric flows. Curves 3 are the phase-equilibrium lines (2.3) plotted in coordinates S_0/R, T_C; for nitrogen [17] A = 9.784, B = 359.1, S'/R = 14.66; for argon [18] A = 9.121, B = 359.3, S'/R = 15.85. The temperature is in °K and p is in Pa.

The experiments in nitrogen cover the range $p_0 = (8\text{-}350)\cdot 10^5$ Pa, T_0 = 290-1730°K, and those in argon cover the range $p_0 = (0.3\text{-}137)\cdot 10^5$ Pa, T_0 = 214-350°K.

Expermental data for water vapor (Fig. 5) have been obtained in the range $p_0 = (0.37\text{-}32)\cdot 10^5$ Pa, T_0 = 370-520°K. The slightly greater spread of the points in comparison with the previous cases can be attributed to the variation of the coefficient β, which corrects the nucleation energy in the equation for the nucleation rate. As was shown in [5], for water vapor β is a complex function of the static parameters and lies between 0.75 and 2.9. The upper and lower experimental points in Fig. 5 correspond to these values of β. Nevertheless, the experimental points from [14] and several points from [16], for which β differs from unity by not more than ±5%, lie satisfactorily on the theoretically-calculated curve for β = 1 (line 1). The phase-equilibrium curve in Fig. 5 (line 2) is plotted from Vargaftik's data [18].

The comparatively small spread of the points relative to curves 1 and 2 and their similar nature in Figs. 3-5 indicate that the assumptions made above are valid, and that this method can be used to treat the experimental data in a wide range of parameters.

The curves in Figs. 3 and 4 can be used, in turn, to estimate the temperature at the Wilson point and the position of the condensation shock in the nozzle. For this we need to calculate the value of the entropy S_0 (2.2), determine T_C from the curve 3, then calculate the parameter Σ (2.4), and from curve 1 (or 2) find the corresponding value of T_V/T_C. Such an estimate in a random case will be reliable if the following two restrictions, which are sufficient and are directly derived from the spontaneous condensation model used above, are satisfied. One of them requires equilibrium of the gas expansion up to the saturation state [19], and the other requires the quasisteadiness of nucleation in the vicinity of the hypothetical Wilson point [13]. The conditions of the experiments in [7-10] correspond very closely to these restrictions.

3. As is known, the self-oscillating state is realized in the case of spontaneous condensation at low Mach numbers [3]. If the amount of phase transition heat brought to the flow exceeds the limit for the given M number, an unsteady shock wave is formed. In moving upstream this shock wave passes through the region of intense nucleation and rules out the possibility of a phase transition. The shock wave, retaining its translational motion, becomes weaker under the action of rarefaction waves and degenerates, thus opening the way for the formation of a condensation shock and repetition of the cycle. Following [3, 4], it can be shown that the position of this condensation shock is also given by a relation of the form (2.1)

$$\Phi_2(T_V, S_0) = \varphi_4(l_*, T_0),\ \varphi_4(l_*, T_0) \doteq \frac{l_*}{\left(2i\,\frac{\varkappa}{\varkappa+1}RT_0\right)^{1/2}},\ l_* = (R_* h_*)^{1/2}, \tag{3.1}$$

where R_* is the radius of curvature of the nozzle wall in the region of the critical cross section; l_* is the characteristic linear scale of such flows.

For self-oscillating regimes the stagnation parameters usually correspond to a state of low superheating. Hence, as the scales of the gasdynamic quantities we select their values in the critical cross section of the nozzle. Then, from the conditions I_2 = idem, I_3 = idem, and I_4 = idem we successively derive the conditions

$$l_* = \text{idem},\ \rho_* = \text{idem},\ T_* = \text{idem},\ I_1 = \text{idem}. \tag{3.2}$$

If the flow to the critical cross section remains isentropic (S_0 = const), then it follows from (3.2) and condition Sh = idem that flows with fixed stagnation parameters, scale l_*, and oscillation period τ will be similar, i.e., the relations

$$p_0 = \text{idem},\ T_0 = \text{idem},\ l_* = \text{idem},\ \tau = \text{idem}. \tag{3.3}$$

will be conserved. In this case the possibility of combining the condition τ = idem with the other conditions from (3.3) has to be established independently.

To determine the frequency characteristics we carried out parametric calculations of self-oscillating nitrogen flow regimes in nozzles of different scale by the method of [3].

Thus, the crosses in Fig. 6 mark the values of Sh; the ratio T_V/T_0 is the argument. The continuous lines connect the values of Sh obtained for a nozzle with $l_* = 10^{-2}$ m, the dashed lines do the same for a nozzle with $l_* = 10^{-1}$ m, and the dot-dash lines for a nozzle with $l_* = 1$ m. The families of curves 1-3 and curve 4 correspond to constant values of the entropy S_0, equal to $5.24 \cdot 10^3$, $5.09 \cdot 10^3$, $4.99 \cdot 10^3$, and $5.17 \cdot 10^3$ J/(kg·°K). The relations $Sh(T_V/T_0)$ when S_0 = idem are, in fact, linear; the gradient of the straight lines is slightly altered. In addition, the effect of the nozzle scale also decreases monotonically with reduction of entropy. The presented data can be generalized by the relation

$$Sh = 16.55 T_V/T_0 + (7.699 - 1.554 \cdot 10^{-3} S_0) \log \varphi_4 + 2.018 \cdot 10^{-2} S_0 - 23.973, \quad (3.4)$$

the mean error of which does not exceed 20% (S_0, J/(kg·°K); φ_4, sec).

It can be deduced from relations (3.1) that when condition (3.3) is fulfilled the equalities φ_4 = idem and T_V = idem and, according to (3.4), the condition τ = idem will also be fulfilled. In other words, fulfillment of the conditions p_0 = idem and T_0 = idem ensures the reproduction of the frequency characteristics of a self-oscillating flow in nozzles with the same value of l_*.

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STABILITY OF RELATIVE MOTION OF PHASES IN TWO-PHASE FLOWS

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The stability of homogeneous states of a two-phase medium in relation to small disturbances (the problem of the correctness of the Cauchy problem for equations of two-phase media) is examined. We show that a consideration of the effect of particle (bubble) diffusion caused by relative motion of the phases is of fundamental importance. The pressure in the disperse phase is a subsidiary factor. The critical stability loss curve is obtained.

The problem of stability of two-phase media has been examined in many papers [1-5]. Existing theories predict a short-wave instability of sedimenting suspensions, fluidized beds, and layers of liquid with bubbles. This instability should lead to the rapid appearance of inhomogeneities within the medium and to the practical unattainability of the homogeneous state. Contradictory to theory, however, manifestly stable states are obtained in experiments [4]. Stability of a liquid with bubbles has been obtained only in [6, 7]. In [6] stability was secured by the action of electrical forces. In the problem of thermocapillary motion in a gas–liquid mixture stability in the short-wave region is obtained by bubble diffusion [7].

1. Equations and Method of Solution

The equations for the change of momentum and conservation of mass of a two-phase medium have the form [1]

$$\varepsilon\rho d\mathbf{v}/dt = \varepsilon\rho\mathbf{g} - \Delta p - \operatorname{div}\mathbf{P}_1 - c\mathbf{F},\ \rho_s\, d\mathbf{u}/dt = \rho_s\mathbf{g} - (1/c)\operatorname{div}\ \mathbf{P}_s + \mathbf{F}; \tag{1.1}$$

$$\partial c/\partial t + \operatorname{div} c\mathbf{u} = 0,\ \partial\varepsilon/\partial t + \operatorname{div}\varepsilon\mathbf{v} = 0,\ c + \varepsilon = 1, \tag{1.2}$$

where ρ, ε, $\mathbf{v}$ and ρ_c, c, $\mathbf{u}$ are the densities, volume concentrations, and densities, respectively, of the carrier and disperse phases; g is the acceleration of gravity. The force of phase interaction F depends, in particular, on the relative velocity of the phases $\mathbf{w} = \mathbf{u} - \mathbf{v}$. The dispersed particles are assumed to be spheres of the same radius R.

At low Reynolds numbers (Re = wR/ν) the force of phase interaction, with due allowance for particle diffusion [8], has the form

$$\mathbf{F} = -\rho\mathbf{g}' + \mathbf{F}^*,\ \mathbf{F}^* = \mathbf{F}_0^* + \mathbf{F}_1^*, \tag{1.3}$$
$$\mathbf{F}_0^* = -\frac{\mu G}{R^2}\mathbf{w},\ \mathbf{F}_1^* = -\frac{\mu}{R^2}\frac{\partial Gw}{\partial w}c^{-1}\mathbf{D}\nabla c,$$

where $\rho\mathbf{g}'$ is the effective repulsive force, $\mathbf{g}' = \mathbf{g} - d\mathbf{v}/dt$; $\mathbf{F}_0^*$ is the viscous resistance force; $\mathbf{F}_1^*$ is the small contribution due to diffusion; G is a dimensionless number; $\mathbf{D}$ is the diffusion tensor:

$$D_{ij} = R|w|(f_{\perp}\delta_{ij} + (f - f_{\perp})w_iw_j/w^2). \tag{1.4}$$

The particle pressure in the medium is given by the tensor

$$(P_s)_{ij} = \rho_s w^2 S_{\perp}\delta_{ij} + \rho_s(S - S_{\perp})w_iw_j. \tag{1.5}$$

At finite Re the coefficient G in (1.3) depends on Re and is connected with the drag coefficient: C_W = G/Re. In this case it is essential to take into account the added-mass effect, which, following [1], we write in model form

$$\mathbf{F}_m = (1/2)\rho(d\mathbf{v}/dt - d\mathbf{u}/dt). \tag{1.6}$$

The fluctuations of the acceleration of the liquid must also be taken into account:

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 83-90, January-February, 1982. Original article submitted June 19, 1980.

 0021-8944/82/2301-0076$07.50